INFINITE SERIES P TEST AND DIRECT COMPARISON TEST

P TEST STATEMENT

You are given an infinite series where p is any real number. If the series converges. If then the series diverges.

COMPARISON TEST STATEMENT

You are a given an infinite series with . Let be a known series of positive terms, of your own choosing, that converges. If for all n, then the original series converges .

Alternately, let be a known series of positive terms, of your own choosing, that diverges. If for all n, then the original series diverges .

The test can be relaxed a little bit. Instead of the inequalities holding for all n, they must hold for some N whenever n > N. So the inequality between an and bn does not have to hold for all n – but it must kick in totally for large values of n.

The power p = 2 and is greater than 1 so the series converges

The power p = 3 and is greater than 1 so the series converges

The power p = 4 and is greater than 1 so the series converges

The power p = 1.1 and is greater than 1 so the series converges

The power p = 1 so the series diverges

The power p = 1/2 and is less than 1 so the series diverges

The power p = 1/3 and is less than 1 so the series diverges

The power p = -2 and is less than 1 so the series diverges

The power p = 0.991 and is less than 1 so the series diverges

and let

for all n. Take reciprocals of this.

for all n

This means that

Since converges, then by the comparison test also converges.

and let

We know

Divide both sides by

for all n

This means that

The series is a geometric series and converges to 1

By the comparison test the series must converge.

for all n and let

We have the inequality for all positive n

Divide both sides of the inequality by 2n

This means that

The series on the right is a geometric series and converges to 1.

By the comparison test, the original series converges. converges

for n > 1 and let for n > 1

We have the inequality for n > 1

Take the reciprocal. We get

Multiply both sides by n (where n>1)

So the series have the inequality

The series on the right is the harmonic series and is known to diverge. Its divergence is established by the p test or the integral test.

By the comparison test, the original series diverges. diveges



and let both greater than zero for n > 2

We have the inequality: for n > 2

Take square roots: for n > 2

Take reciprocals: for n > 2

This gives the inequality

The series on the right is the harmonic series (with the first term missing). The starting of a series does not affect convergence so the series on the right diverges.

By the comparison test, the original series diverges. diverges



for k > 1

From the above product we can see the following inequality:

for k > 1

Taking reciprocals we have for k > 1

We have the following inequality for the series:

Change the index on the second series and let n = k-1

The series is known to converge by either the p test or the integral test.

By the comparison test, the integral on the left must converge. Converges.

and let both positive for n > 0

We start with the inequality for n > 0

Taking reciprocals we get: < for n > 0

This gives the following inequality for series

The second series is known to converge by the p test or the integral test.

The series must converge by the comparison test.

and for n > 2 (both terms are positive)

We start with the inequality that for n > 2

Divide by sides by n3

for n > 2

This gives the inequality for the series

Since the series on the right converges by p test, the series on the left must converge by comparison test.

converges

We start with the inequality for n > 0

Divide both sides by n+2 where n > 0

where n > 0

This gives us the following inequality for the series:

The integral on the right diverges by the integral test.

The series on the left diverges by the comparison test. diverges

This is a bit of a nasty one so I am going to go directly to an inequality that will get us home:

for n > 0 where n is an integer

This yields

The series on the right converges by p test. The series on the left converges by comparison test.

converges



We start with the following inequality:

for where n is an integer

Now take reciprocals:

for where n is an integer.

Take positive square roots of both sides:

for where n is an integer

We can now form series:

The series on the right is the harmonic series and diverges by p test or integral test.

The series on the left must diverge by comparison test.

diverges

We start with the inequality for where n is an integer

Take square roots of both sides: for

Take reciprocals to get:

Now form series:

The series on the right converges by the p test. So the series on the left must converge by comparison test.

converges

This one is a bit tricky.

We start with the inequality: for

Multiply both sides by -1: for

Add 2k2 to both sides:

Take reciprocals:

Form the series:

The series on the left converges by the p test. So the series on the right converges by comparison test.

converges.